NED UNIVERSITY OF ENGINEERING & TECHNOLOGY, KARACHI
FIRST YEAR (COMPUTER SCIENCE AND INFORMATION TECHNOLOGY)
ANNUAL EXAMINATION 2007
BATCH 2006-07

Time: 3 Hours **Dated: 18-10-2007**
Max. Marks: 80

DIFFERENTIAL & INTEGRAL CALCULUS
(MS-171)

Instructions: 1. Attempt any **FIVE** questions.
2. All questions carry equal marks.

Q1a): If $f(x)=\begin{cases}\dfrac{x^2-9}{x+3}, & x\neq 3\\ k & x=3\end{cases}$ [4]

Find k so that $f(-3)=\lim_{x\to -3} f(x)$

b): Find the limit [12]

i) $\lim_{x\to 1}(2-x)^{\tan[(\pi/2)x]}$ ii) $\lim_{x\to 0}(\operatorname{cosec} x-1/x)$ iii) $\lim_{x\to 0}\dfrac{e^x-1}{\sin x}$

Q2: Sketch the graph by using x-intercept, y-intercept, intervals of increase and decrease, stationary points, concavity, inflection points and horizontal and vertical asymptotes. [16]

$$f(x)=\frac{x^2}{x^2-1}$$

Q3a) : Drive the reduction formula for $\int Sin^n x dx$ and evaluate $\int \sin^8 dx$ by the formula. [8]

b) : Compute $\int_0^{\frac{\pi}{2}} Cos^{12} x dx$ by using walli's formula [8]

Q4a): Two roads intersect at right angles. Car A moving on one of the roads, approaches the intersection at 25km/hr and car B moving on the other road, approaches the intersection at 30km/hr. At what rate is the distance between the cars changing when A is 0.3km from the intersection and B is 0.4km from the intersection? [6]

b): If $T=x^2y-xy^3+2$; $x=r\cos\theta$, $y=r\sin\theta$. Find $\dfrac{\partial T}{\partial r}$ and $\dfrac{\partial T}{\partial \theta}$ [5]

c): Let f be a differentiable function of three variables and suppose that $w=f(x-y,y-z,z-x)$, then show that $\dfrac{\partial w}{\partial x}+\dfrac{\partial w}{\partial y}+\dfrac{\partial w}{\partial z}=0$ [5]

P.T.O

Q5a): If $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$ then show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$ [8]

b): Find all relative maxima, relative minima and saddle point, if any [8]

$f(x, y) = x^2 + xy + y^2 - 3x$

Q6a): For gama integral $\Gamma(x)$ prove that $\Gamma(1) = 1$ and $\Gamma(n+1) = n!$ [8]

b): By using Beta and Gama integrals find $\int_0^{\pi/4} \sin^4 2x \cos^6 2x dx$ [8]

Q7a): Evaluate $\iint_R x(1 + y^2) dA$; where R is the region in the first quadrant enclosed by $y = x^2, y = 4$ and $x = 0$ [8]

b): By the double integral find the area of the region enclosed by $y = \sin x$ and $y = \cos x$, for $0 \le x \le \pi/4$ [8]

Q8a): If z_1, z_2 and z_3 are complex numbers then prove that $|z_1 + z_2| \le |z_1| + |z_2|$ [8]

b): Find all the roots of the equation $x^7 + 1 = 0$ [8]

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CALCULUS – (MS-156)

Do any five

Q1a) Find the 6^{th} root of -1 + I &64i [8]

b) Prove that $Cos^4\theta + Sin^4\theta = \frac{1}{4}(Cos4\theta + 3)$

Q2a): If z_1, z_2 and z_3 are complex numbers then prove that $|z_1 + z_2| \le |z_1| + |z_2|$ [8]

b): Find all the roots of the equation $x^7 + 1 = 0$ [8]

Q3): Find the limit [16]

i) $\lim_{x\to 0}\frac{e^x - 1}{\sin x}$ ii) $\lim_{x\to 0}(e^x + x)^{1/x}$ iii) $\lim_{x\to 0}\frac{Sin2x}{Cos3x}$ iv) $\lim_{x\to 0}(\frac{1}{x} - \frac{1}{e^x - 1})$

Q4a) :Integrate the following

i) $\int Sin^n x dx$. ii) $\int Co\sec^n x dx$

iii) $\int_0^{\pi/4} \tan^8 x dx$ iv) $\int_0^{\infty} e^{-kt^n} dt$ [16]

Q5a): Find the arc length of $r = a(1 - Cos\theta);\ 0 < \theta < \pi$ [8]

b): Find all relative maxima, relative minima and saddle point, if any [8]

$f(x, y) = x^2 + y^2 + \frac{2}{xy}$

Q6a): State and prove the Leibnitz theorem [8]

b): Find all asymtotes of $x^2 y + y^2 x = a^2$ [8]

Q7a): If $T = x^2 y - xy^3 + 2;\ x = r\cos\theta,\ y = r\sin\theta$. Find $\frac{\partial T}{\partial r}$ [3]

b): Let f be a differentiable function of three variables and suppose that $w = f(x - y, y - z, z - x)$, then show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$ [5]

P.T.O

c): If $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$ then show that [8]

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

Q8a): For gama integral $\Gamma(x)$ prove that $\Gamma(1) = 1$ and $\Gamma(n+1) = n!$ [8]

b): By using Beta and Gama integrals find $\int_0^{\pi/4} \sin^4 2x \cos^6 2x dx$ [8]

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