NED UNIVERSITY OF ENGINEERING & TECHNOLOGY, KARACHI
FIRST YEAR (COMPUTER SCIENCE AND INFORMATION TECHNOLOGY)
ANNUAL EXAMINATION 2008
BATCH 2007-08

Time: 3 Hours

Dated: 15-10-2008
Max. Marks: 75/80

DIFFERENTIAL & INTEGRAL CALCULUS
(MS-171)

Instructions: (i) Attempt **FIVE** questions in all.
(ii) Selecting at least **TWO** questions from each section:
(iii) All questions carry equal marks.

SECTION - A

Q # 1 (a) Discuss the **continuity** of the following functions at the point $x=-1,0,2$ **(8/8)**

$$f(x)=\begin{cases} x^3-1 & x\geq 0 \\ x^2 & x=0 \\ x^3+1 & x\leq 0 \end{cases}$$

(b) If **A={x | x ∈ N, for all 1 ≤ x ≤ 4}**, Find the following relation define on A × A : **(7/8)**

1) $R_1=\{(x,y)\mid(x,y)\in A\times A;\ x+y=\text{even}\}$
2) $R_2=\{(x,y)\mid(x,y)\in A\times A;\ y=x^2\}$
3) $R_3=\{(x,y)\mid(x,y)\in A\times A;\ x^2+y^2=16\}$

Q # 2 (a) Using **Leibnitz Theorem**, compute $\frac{d^6y}{dx^6}$ of $y=e^{x^2}(x^4+x^3+1)$ **(8/8)**

(b) It is required to construct the rectangular box that will hold the volume of **4 m^3** having square base and open top. Compute the suitable dimension such that material required is minimum. **(4/4)**

(c) If the body of mass "m" is situated at a height "h". This height is noted **10.05 meters** instead of **10 meters**. Then find the error in potential energy (P_E) of the body due to height . Take $P_E = mgh$ and **g = 10 m/s^2**. **(3/4)**

Q No. 3 (a) Locate and discuss the nature of the **stationary point** of the curve **(8/8)**

$$F(x,y)=x^4+y^4-36xy$$

(b) Find the limit of the following: **(4/4)**

(i) $\lim\limits_{X\to 0}\frac{e^x-1}{x^2}$ (ii) $\lim\limits_{X\to 1}\frac{x^5-1}{x^4-1}$

(c) Plot the following curve: **(3/4)**

$$f(x)=Sgn(x)-1,\ D_f=[-3,3)$$

P.T.O

Q No. 4 (a) Find the curvature of the curve $x^3 + y^3 = 3ax\,y$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. (8/8

(b) Find root of the equation by using **Newton Raphson Method** of the equation $2x^3 + 2x^2 - 1 = 0$ lies in interval $[0, 1]$. Take mid point of this interval as x_o and **iterate six times** : (7/8

SECTION - B

Q No. 5 (a) Using Reduction formulae for trigonometric function, find

$$\int \tan^7 \theta \, d\theta$$

(8/8)

a) By using **Beta and Gamma integral**, evaluate the following integral

(i) $\int_0^\infty \frac{e^{-x}}{x^{-4}}\,dx$ (ii) $\int_0^1 - x^4 (x-1)^3\,dx$ (7/8

Q No. 6 (a) Show that $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right) = \int_0^{\pi/2} \sin^m \theta \cos^n \theta \, d\theta$ for $m, n \in N^+$ (7/8

(b) $\beta(m,n) = \beta(m,n+1) + \beta(m+1,n)$ (4/4

(c) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (4/4

Q No. 7 (a) Show that if $\tan(\theta + i\beta) = \tan\alpha + i\sec\alpha$, then

$$e^{2\beta} = \pm \mathrm{Cot}(\alpha/2)$$

(8/8)

(b) Expand **Sin 4θ** in the powers of Sin θ Cos θ: (4/4)

(c) If $X = \mathrm{Cos}\alpha + i\,\mathrm{Sin}\alpha$ and $Y = \mathrm{Cos}\theta + i\,\mathrm{Sin}\theta$ then show that

$$X^m Y^n - X^{-m} Y^{-n} = 2i\,\mathrm{Sin}(m\alpha + n\alpha)$$

(3/4)

Q No. 8 (a) Show that $\vec{r}\,(x,y,z) = 2xyz\,i + (x^2z + 2y)\,j + x^2y\,k$ is irrotational: (8/8

a) Find the directional derivative of $\varnothing(x,y,z) = xy^2 + yz^3$ at $(1,-1,1)$ in direction of $\vec{A} = 3i + j - 2k$. (4/4)

b) Compute $\nabla \cdot r^3 \vec{r}$,where $r = \sqrt{x^2 + y^2 + z^2}$ $\vec{r} = x\,i + y\,j + z\,k$ (3/4

— X —

NED UNIVERSITY OF ENGINEERING & TECHNOLOGY, KARACHI
FIRST YEAR (COMPUTER SCIENCE AND INFORMATION TECHNOLOGY)
ANNUAL EXAMINATION 2008
(FOR REPEATERS)

Time: 3 Hours | **Dated: 15-10-2008**
Max. Marks: 80

CALCULUS – (MS-156)

Instructions: 1. Attempt any **FIVE** questions.
2. All questions carry equal marks.

Q1. (a) Using De Moivre's theorem, Prove that (8)

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(b) Evaluate $|Z|$ for the complex number $Z = \frac{(3+4i)(2i-1)}{(-1-i)(3-i)}$. (8)

Q2. (a) If $\tan(A+iB) = x+iy$ then show that $x^2+y^2+2x\cot 2A = 1$ (8)

and also $x^2+y^2-2y\coth 2B = -1$.

(b) Show that $\tanh^{-1} z = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$. (8)

Q3. (a) If $y = e^{m\sin^{-1}x}$ then show that $(1-x^2)y^{(n+2)} - (2n+1)xy^{(n+1)} - (m^2+n^2)y^{(n)} = 0$ (8)

by using Leibnitz's theorem of differentiation.

(b) Expand e^{2x} in power of $(x-1)$, using Taylor's theorem. (8)

Q4. (a) State and prove Euler's theorem for the function of one variable. (8)

(b) If $z = \ln(x^2+y^2)$ then show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$. (8)

Q5. (a) Find the radius of curvature of the curve $x = 3t^2, y = 3t - t^3$ at $t = 1$. (8)

P.T.O

(b) Find the coordinates of center of curvature of the curve $x^3 + xy^2 - 6y^2 = 0$ at the point $(3,3)$. (8)

Q6. (a) Evaluate the following by using reduction formula (8)

$$\int (\sin x)^6 (\cos x)^2 \, dx$$

(b) Determine a reduction formula for $\int \cos^n x \, dx$. (8)

Q7. (a) Find the max and min also find the point of inflection for the curve (8)

$$f(x) = 2x^3 - 15x^2 + 36x + 10.$$

(b) Find the equation of asymptotes of the curve $x^2 y^2 = 12(x-3)$ (8)

Q8. Find the limit of the following: (16)

(i) $\lim_{x \to 0} \frac{e^{2x} - 1}{\sin 2x}$ (ii) $\lim_{x \to 0} (e^x + x)^{1/x}$ (iii) $\lim_{x \to 0} \frac{\sin 3x}{\cos 4x - 1}$

(iv) $\lim_{x \to \infty} \frac{x^2 + 2x - 1}{(x+1)(x+2)}$

................................GOOD LUCK..